BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
(BULLETIN OF THE POLYTECHNIC INSTITUTE OF IAȘI)
Published by "Gheorghe Asachi" Technical University of Iasi
Volume 67(71), No. 4, 2021
Section Mathematics. Theoretical Mechanics. Physics

DOI: 10.2478/bipmf-2021-0022

ACOUSTIC BLACK HOLE GENERATED BY A CLUSTER OF OSCILLATING BUBBLES

BY

ZOLTAN BORSOS[1,*], ION SIMACIU[2], GHEORGHE DUMITRESCU[3] and VIOREL DRAFTA[4]

[1]Petroleum-Gas University of Ploieşti, Romania
[2]Retired lecturer, Petroleum-Gas University of Ploieşti, Romania
[3]High School Toma N. Socolescu, Ploiești, Romania
[4]Independent researcher

Received: December 3, 2021
Accepted for publication: December 30, 2021

Abstract. The paper demonstrates that a liquid becomes inhomogeneous in interaction with an oscillating bubbles cluster. Because the refractive index of the liquid around the cluster has a radial variation, a spherical acoustic lens is generated. Under certain restrictive conditions, the acoustic lens behaves like an acoustic black hole.

Keywords: cluster of oscillating bubbles; acoustic black hole; acoustic interaction.

1. Introduction

In previously published papers (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020; Simaciu *et al.*, 2018b) it is demonstrated that a liquid in the presence of a packet of spherical acoustic waves or an oscillating bubble behaves like an acoustic lens.

In this paper, we analyze the same phenomenon generated in liquid by a cluster of oscillating bubbles.

---

*Corresponding author; *e-mail*: borzolh@upg-ploiesti.ro
© 2021 Zoltan Borsos et al.
This is an open access article licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0).

In the second section, we deduce the expression pressures in the liquid around a cluster that oscillates under the action of an acoustic wave. The refractive index of the liquid around the cluster whose expression is deduced in the third section has properties similar to those of the refractive index of an oscillating bubble (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020; Simaciu *et al.*, 2018b). Because the refractive index of the liquid around the cluster has a radial variation, a spherical acoustic lens is generated.

The fourth section demonstrates that the acoustic lens associated with the cluster becomes an acoustic black hole (dumb hole or sonic black hole) if the focal length $f(r)$ of the acoustic lens is equal to the minimum distance r to the center of the cluster, at which the deflected acoustic wave passes.

The condition is that the radius of the cluster, compressed under the action of acoustic forces (Wang and Cheng, 2013; Simaciu *et al.*, 2019c), is smaller than the acoustic radius (radius of the acoustic black hole) associated with the oscillating cluster.

The paper is completed with conclusions and discussions in the fifth section.

2. Sound pressure near the oscillating cluster

In order to deduce the mathematical relation for the sonic / acoustic pressure around an oscillating cluster, we consider that this system having radius R_c consists of N bubbles with identical radii ($R_{0i} = R_0, i = 1,2,\ldots,N$). The bubbles of the cluster are oscillated by a plane excitation wave having the pulsation ω. The mathematical relation for the sonic pressure has the expression similar to the expression of the pressure around an oscillating bubble (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020).

$$p'_N(r,t) \cong \frac{\rho \ddot{V}_N}{4\pi r} = \frac{\rho R_N}{r}\left(2\dot{R}_N^2 + R_N\ddot{R}_N\right) \cong \frac{\rho R_N^2 \ddot{R}_N}{r} \cong \frac{\rho R_0^3 \omega^2 x_N}{r}. \qquad (1)$$

The phenomenon of the coupling of the oscillations of the bubbles in the cluster determines that the dimensionless amplitude (elongation) of the oscillations is (Wang and Cheng *et al.*, 2013; Simaciu *et al.*, 2019c)

$$x_N = a_N \cos(\omega t + \varphi_N) \qquad (2)$$

with

$$a_N = \frac{A}{\rho R_0^2\left[\left(\omega^2(1+N_c)-\omega_0^2\right)^2 + 4\beta^2\omega^2\right]^{\frac{1}{2}}}, \quad \varphi_N = arctan\frac{2\beta\omega}{\omega^2(1+N_c)-\omega_0^2}. \qquad (3)$$

Substituting in equation (1) equations (2) and (3) is obtained

$$p'_N(r,t) \cong \frac{R_0\omega^2 A\, cos(\omega t+\varphi_N)}{r\left[\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2\right]^{\frac{1}{2}}}. \tag{4}$$

We approximate the external, $r \geq R_c$, sound pressure to the cluster by the relationship

$$p'_{Ncl}(r,t) \cong \frac{NR_0\omega^2 A\, cos(\omega t+\varphi_N)}{r\left[\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2\right]^{\frac{1}{2}}}. \tag{5}$$

Expressions of the amplitude and the phase (3), at the resonance, $\omega = \omega_{0Nr} = \frac{\omega_{0r}}{\sqrt{1+N_c}} < \omega_{0r}$, are:

$$a_{N,res} = \frac{A}{2\rho R_0^2 \beta_{N,res}\omega_{0N,res}}, \quad \varphi_{N,res} = arctan\,\infty = \frac{\pi}{2}. \tag{6}$$

Also in the condition of resonance, the equations (4) and (5) acquire the expressions:

$$p'_{N,res}(r,t) \cong \frac{R_0\omega_{0N,res}A\, sin(\omega_{0N,res}t)}{2r\beta_{N,res}}, \tag{7}$$

$$p'_{Ncl,res}(r,t) \cong \frac{NR_0\omega_{0N,res}A\, sin(\omega_{0N,res}t)}{2r\beta_{N,res}}. \tag{8}$$

If we approximate the damping coefficient, $\beta_{N,res}$, through the relations

$$\beta_{N,res} = \beta_{0Nac,res} + \beta_{0N\mu,res} \cong \beta_{0Nac,res} = \frac{\omega_{0N,res}^2 R_0}{2u}, \tag{9}$$

With this approximation, equations (7) and (8) become

$$p'_{N,res}(r,t) \cong \frac{uA\, sin(\omega_{0N,res}t)}{r\omega_{0N,res}} = \frac{uA\sqrt{1+N_c}\, sin(\omega_{0N,res}t)}{r\omega_0}, \tag{10}$$

$$p'_{Ncl,res}(r,t) \cong Np'_{N,res}(r,t) = \frac{N\sqrt{1+N_c}uA\, sin(\omega_{0N,res}t)}{r\omega_0},\ r > R_c. \tag{11}$$

We have shown, according to equations (10) and (11) that the sonic pressure of the fluid around the oscillating cluster has spherical symmetry.

3. The refractive properties of the fluid around the cluster

To obtain the expression of the fluid refractive index in the presence of the oscillating cluster we use the relation (10) from the paper "Acoustic Lens Associated with a Radial Oscillating Bubble" (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020) with $\delta p = p'_{Ncl}(r,t)$

$$\langle n_{acl}(p)\rangle_t \cong 1 + \frac{\zeta(3\zeta+1)}{2(2\zeta+1)^2}\frac{\left\langle \left(p'_{Ncl}(r,t)\right)^2\right\rangle_t}{p_0^2}. \qquad (12)$$

By replacing relations (5) and (11) in relation (12), it becomes:

$$\langle n_{acl}(p)\rangle_t \cong 1 + \frac{N^2\zeta(3\zeta+1)R_0^2}{4(2\zeta+1)^2 r^2}\frac{A^2\omega^4}{p_0^2\left[\left(\omega^2(1+N_c)-\omega_{0r}^2\right)^2+4\beta_r^2\omega^2\right]}, \qquad (13)$$

$$\left\langle n_{acl,res}(p)\right\rangle_t \cong 1 + \frac{N^2(1+N_c)\zeta(3\zeta+1)}{4(2\zeta+1)^2 r^2}\frac{A^2u^2}{p_0^2\omega_0^2}. \qquad (14)$$

Equations (13) and (14) highlight the dependence, with spherical symmetry of the refractive index of the fluid around the cluster, on the position vector r. This property is specific to a spherical acoustic lens.

Acoustic lens corresponding to the oscillating cluster has a positive focal length depending on the impact radius (Feynman *et al.*, 1964):

$$\frac{1}{f_{cl}} = \frac{2}{r}\left(\langle n_{acl}\rangle_t - 1\right) = \frac{N^2\zeta(3\zeta+1)}{2(2\zeta+1)^2}\frac{R_0^2}{r^3}\frac{A^2}{p_0^2}\frac{\omega^4}{\left[\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2\right]} \qquad (15)$$

and

$$\frac{1}{f_{cl,res}} = \frac{2}{r}\left(\left\langle n_{acl,res}\right\rangle_t - 1\right) = \frac{N^2(1+N_c)\zeta(3\zeta+1)}{2(2\zeta+1)^2}\frac{A^2}{p_0^2}\frac{u^2}{\omega_0^2 r^3} \qquad (16)$$

or

$$f_{cl} = \frac{2(2\zeta+1)^2}{N^2\zeta(3\zeta+1)}\frac{r^3}{R_0^2}\frac{p_0^2}{A^2}\frac{\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2}{\omega^4} > 0 \qquad (17)$$

and

$$f_{cl,res} = \frac{2(2\zeta+1)^2}{N^2(1+N_c)\zeta(3\zeta+1)}\frac{p_0^2}{A^2}\frac{\omega_0^2 r^3}{u^2} > 0. \qquad (18)$$

It turns out that this acoustic lens being convergent deflects the trajectory of the acoustic waves towards the cluster.

4. Acoustic black hole associated with a cluster

The trajectory of a plane acoustic wave becomes a circle if the focal length of the acoustic lens is equal to the minimum distance (the impact distance for the trajectory of the acoustic wave),$r_m > R_c$, from the center of the cluster at which the wave passes

$$f(r_m) = r_m. \tag{19}$$

By replacing equations (17) and (18) in the restrictive relation (19), the acoustic lens becomes an acoustic black hole with an acoustic radius equal to the minimum distancer_m:

$$r_{acl} = R_0 \frac{N\sqrt{\zeta(3\zeta+1)}}{\sqrt{2}(2\zeta+1)} \frac{A}{p_0} \frac{\omega^2}{\sqrt{\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2}} = N r_{abcl}, \tag{20}$$

$$r_{acl,res} = \frac{N\sqrt{1+N_c}\sqrt{\zeta(3\zeta+1)}}{\sqrt{2}(2\zeta+1)} \frac{Au}{p_0\omega_0} \cong \frac{N\sqrt{N_c}\sqrt{\zeta(3\zeta+1)}}{\sqrt{2}(2\zeta+1)} \frac{Au}{p_0\omega_0} = N r_{abcl,res}. \tag{21}$$

These acoustic radii, the acoustic radius of a coupled bubble$r_{abcl}$and the resonant coupled bubble$r_{abcl,res}$, are different from the acoustic radius of an uncoupled oscillating bubble (relation (17) in the papers (Simaciu I. *et al.*, 2018a; Simaciu I. *et al.*, 2020):

$$r_{abcl} = R_0 \frac{\sqrt{\zeta(3\zeta+1)}}{\sqrt{2}(2\zeta+1)} \frac{A}{p_0} \frac{\omega^2}{\sqrt{\left(\omega^2(1+N_c)-\omega_0^2\right)^2+4\beta^2\omega^2}}, \tag{22}$$

$$r_{abclr} = \frac{\sqrt{N_c}\sqrt{\zeta(3\zeta+1)}}{\sqrt{2}(2\zeta+1)} \frac{Au}{p_0\omega_0}. \tag{23}$$

Since, conformable to the equation (25) of the paper (Simaciu I. *et al.*, 2019c), $N_c \cong N\left(\frac{3R_0}{(2R_c)}\right) >> 1$, Eqs. (21) and (23) becomes

$$r_{acl,res} = \frac{N^{\frac{3}{2}}\sqrt{3R_0}\sqrt{\zeta(3\zeta+1)}}{2\sqrt{R_c}(2\zeta+1)} \frac{Au}{p_0\omega_0} = N r_{abcl,res} \tag{24}$$

and

$$r_{abcl,res} = \frac{\sqrt{3NR_0}\sqrt{\zeta(3\zeta+1)}}{2\sqrt{R_c}(2\zeta+1)} \frac{Au}{p_0\omega_0}. \tag{25}$$

Replacing the natural angular frequency, $\omega_0 = \left\{3\gamma\left[p_0/\left(\rho R_0^2\right) + \frac{2\sigma}{(\rho R_0^3)}\right] - \frac{2\sigma}{(\rho R_0^3)}\right\}^{\frac{1}{2}} = \left[\frac{p_{eff}}{(\rho R_0^2)}\right]^{\frac{1}{2}}$ (Prosperetti, 1977), in Eqs. (24) and (25), results: $\omega_0 = \left\{3\gamma\left[\frac{p_0}{\rho R_0^2} + \frac{2\sigma}{\rho R_0^3}\right]\left\{-\frac{2\sigma}{(\rho R_0^3)}\right\}^{\frac{1}{2}}$

$$r_{acl,res} = \frac{R_0 N^{\frac{3}{2}}\sqrt{3R_0}\sqrt{\zeta(3\zeta+1)}}{2\sqrt{R_c}(2\zeta+1)}\frac{A}{p_0}\sqrt{\frac{\rho u^2}{p_{eff}}} = N r_{abcl,res}, \tag{26}$$

$$r_{abcl,res} = \frac{R_0\sqrt{3NR_0}\sqrt{\zeta(3\zeta+1)}}{2\sqrt{R_c}(2\zeta+1)}\frac{A}{p_0}\sqrt{\frac{\rho u^2}{p_{eff}}}. \tag{27}$$

Therefore, if the cluster is compressed to a smaller radius than its acoustic radius $R_c \leq r_{acl,res}$, it behaves like an acoustic black hole. This phenomenon was not possible (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020), for a free bubble (not coupled in a cluster). In a situation of equality $R_c = r_{acl,res}$, we deduce an expression of the acoustic radius that is not a function of the radius of the cluster. With this condition the relations (26) and (27) result:

$$r_{acl,res} = NR_0\left[\frac{3\zeta(3\zeta+1)}{4(2\zeta+1)^2}\left(\frac{A}{p_0}\right)^2\frac{\rho u^2}{p_{eff}}\right]^{\frac{1}{3}} = N r_{abcl,res}, \tag{28}$$

$$r_{abcl,res} = r_{ba,res}\left[\frac{6(2\zeta+1)}{\sqrt{\zeta(3\zeta+1)}}\frac{p_0}{A}\sqrt{\frac{p_{eff}}{\rho u^2}}\right]^{\frac{1}{3}} > r_{ab,res},\ p_0 >> A. \tag{29}$$

In relations (28) and (29), the radius $r_{ab,res}$ is given by the equation (17) of the paper (Simaciu *et al.*, 2018a; Simaciu *et al.*, 2020).

The acoustic radius given by relation (28), $r_{acl,res} = N r_{abcl,res}$, is similar to the gravitational radius of a body consisting of N particles having mass m and gravitational radius $r_{gm} \cong \frac{(Gm)}{c^2}$ (Landau *et al.*, 1971)

$$r_{gM} \cong \frac{G(Nm)}{c^2} = N r_{gm}. \tag{30}$$

The resemblance of the phenomena in the acoustic world to those in the electromagnetic world inspires the hypothesis that two acoustically excited clusters can interact attractively. This phenomenon and its consequences will be studied in a later paper.

5. Conclussion an discussions

We have shown that the properties of the fluid around a spherical system of oscillating bubbles are different at different points.

This acoustically excited bubble system behaves like a converging acoustic lens for acoustic waves passing through it.

The acoustically excited bubble system manifests itself as an acoustic black hole if contracted at a radius smaller than its acoustic radius $R_c \leq r_{acl}$.

It is interesting, from a phenomenological point of view, that the acoustic radius of the acoustic black hole corresponding to the cluster is similar to the gravitational radius of a body consisting of N particles having mass m (Simaciu *et al.*, 2019c). This property complements and supports the analogy between the phenomena of the acoustic world and the electromagnetic world.

Extending the analogy between the two “worlds”, the above results highlight both the phenomenon of acoustic wave scattering by the oscillating cluster and the phenomenon of wave absorption when the minimum radius (impact distance) is smaller than the acoustic radius $r_m < r_{aclr}$. The phenomenon of scattering acoustic waves by the cluster generates electro-acoustic forces between clusters similar to those between free bubbles (Simaciu *et al.*, 2017; Simaciu, 2019a). The ultimate goal of the study of phenomena in the acoustic world is to generate an electrically charged, non-point particle model (Bohm and Weinstein, 1948; López, 2020), by analogy with the “particle” model in the acoustic world which is the oscillating bubble. Particle models in the two "worlds" must be compatible with the systems made up of these particles, from the simplest to the most complex, such as stars and the finite universe. For this reason, we have extended the study of acoustic phenomena from two bubbles, in interaction, to bubble clusters with a large number of bubbles in interaction (Simaciu *et al.*, 2019c).

The phenomenon of absorption of sound waves generates an attractive interaction between clusters and therefore gravito-acoustic forces (Simaciu *et al.*, 2019b). These phenomena specific to the cluster systems and their consequences will be studied in a later paper.

REFERENCES

Bohm D., Weinstein M., *The Self-Oscillations of a Charged Particle*, Phys. Rev. **74**, 1789-1798 (1948).

Feynman R.P., Leighton R.B., Matthew S., *The Feynman Lectures on Physics*, Vol. **1**, Massachusetts: Addison-Wesley, Ch. 27.3. (1964).

Landau L.D., Lifshitz E.M., *The Classical Theory of Fields (Volume 2 of A Course of Theoretical Physics)* Pergamon Press, Ch. 11, Sch. 97, p. 284, Eq. (97.13), (1971).

López Á.G., *On an Electrodynamic Origin of Quantum Fluctuations*, Nonlinear Dynamics volume **102**, 621-634 (2020).

Prosperetti A., *Thermal Effects and Damping Mechanisms in the Forced Radial Oscillations of Gas Bubbles in Liquids*, The Journal of the Acoustical Society of America, **61**, *1*, 17-27 (1977).

Simaciu I, Borsos Z., Dumitrescu Gh., Silva G.T., Bărbat T., *Acoustic Scattering-Extinction Cross Section and the Acoustic Force of Electrostatic Type*, arXiv: 1711.03567 (2017).

Simaciu I., Dumitrescu Gh., Borsos Z., *Acoustic Lens Associated with a Radial Oscillating Bubble*, arXiv:1811.08738 (2018a).

Simaciu I., Dumitrescu Gh., Borsos Z., Brădac M., *Interactions in an Acoustic World: Dumb Hole*, Advances in High Energy Physics, Article ID 7265362, Volume 2018 (2018b).

Simaciu I., Borsos Z., Dumitrescu Gh., Silva G.T., Bărbat T., *The Acoustic Force of Electrostatic Type*, Bul. Inst. Polit. Iaşi, s. Matematică. Mecanică Teoretică. Fizică, **65 (69)**, *2*, 17-28 (2019a).

Simaciu I., Dumitrescu Gh., Borsos Z., *Acoustic Force of the Gravitational Type*, arXiv:1905.03622v1 (2019b).

Simaciu I., Dumitrescu Gh., Borsos Z., *Mach's Principle in the Acoustic World*, arXiv:1907.05713 (2019c).

Simaciu I., Dumitrescu Gh., Borsos Z., Drafta V., *Acoustic Lens Associated with a Radial Oscillating Bubble*, Bul. Inst. Polit. Iaşi, s. Matematică. Mecanică Teoretică. Fizică, **66 (70)**, *2*, 9-15 (2020).

Wang C.H., Cheng J.C., *Interaction of a Bubble and a Bubble Cluster in an Ultrasonic Field*, Chin. Phys. B, **22**, *1*, 014304 (2013).

GAURĂ NEAGRĂ ACUSTICĂ GENERATĂ DE UN CLUSTER DE BULE OSCILANTE

(Rezumat)

Lucrarea demonstrează că un lichid devine neomogen în interacțiunea cu un cluster de bule oscilante. Deoarece indicele de refracție al lichidului din jurul clusterului are o variație radială, se generează o lentilă acustică sferică. În anumite condiții restrictive, lentila acustică se comportă ca o gaură neagră acustică.